Optimum Noise Measure Terminations for Microwave Transistor Amplifiers

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Abstract —**The noise performance of the individual stages in a multistage low-noise amplifier can be quantified by means of the noise measure as proposed by Haus and Adler [4]. The minimization of the noise measure of a given active two-port device is of direct interest to the designer of such amplifiers.**

A new means of obtaining circles of constant noise measure in the source reflection coefficient plane is presented here. These circles can be used to determine the value of the minimum noise measure for a given active device and the associated source termination. The validity of the new method has been verified by comparison with results obtained using existing equations for the admittance plane and also by experiment.

I. Introduction

It is well known that the noise figure of a single linear noisy two-port network can be minimized at a unique value of complex source termination [1], and that loci of constant noise figures are circles in the source admittance [2], [5] or reflection coefficient [5] plane. Due to the limitations of present day transistors, the maximum gain and minimum noise figure for a given device cannot be realized simultaneously. This means that the minimum noise figure of a multistage amplifier will not be obtained if each individual stage is designed for a minimum noise figure, due to the influence of stage gain on overall the noise figure as described by Friis [3]. A compromise between stage gain and noise figure must, therefore, be found if the overall noise figure of a multistage amplifier is to be minimized.

Haus and Adler [4] have shown that the minimum noise figure of a cascade of stages is obtained when the first stage exhibits the minimum value of the quantity M defined by

$$M=\frac{F-1}{1-1/G_a} \tag{1}$$

where

M noise measure of the stage,
F noise figure of the stage,
G_a available power gain of the stage.

When designing multistage amplifiers for minimum noise figure, the designer needs to know how M varies with the terminating impedances of the first active device. In particular, one is interested in the terminating impedances which yield the minimum value of M [11].

This paper deals with the analysis of the effects of the source reflection coefficient on noise measure and presents some new equations, describing these effects, which will be of use to workers in this field.

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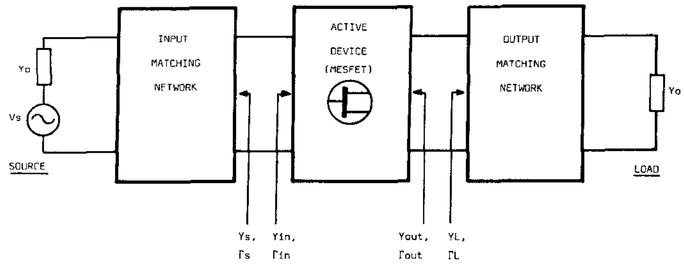

Fig. 1. Generalized representation of a single-stage microwave amplifier using a GaAs MESFET as the active device.

II. Theory

Fig. 1 is a generalized representation of a single-stage microwave amplifier consisting of an active device, such as a GaAs MESFET, with passive input and output matching networks.

The relationship between noise figure F and source admittance Y_s for the linear active device in Fig. 1 is given by [1]:

$$F=F_{\min}+\frac{R_n}{G_s}|Y_s-Y_{\text{on}}|^2 \tag{2}$$

where

$F_{\min}$ minimum noise figure,
R_n equivalent noise resistance parameter,
Y_{on} source admittance which results in $F_{\min}$.

The available gain G_a of the active device may be expressed in terms of the source admittance by an analogous equation [5]

$$\frac{1}{G_a}=\frac{1}{G_{\max}}+\frac{R_{eg}}{G_s}|Y_s-Y_{ms}|^2 \tag{3}$$

where

$G_{\max}$ maximum available gain,
R_{eg} equivalent gain resistance parameter,
Y_{ms} source admittance which results in $G_{\max}$.

Thus, both the noise figure and gain of an active two-port network are explicit functions, dependent on certain parameters, of source admittance. Fukui [5] has used (1)–(3) to show that loci of constant noise measure in the source admittance plane are circles described by the equation

$$|Y_s-Y_m|^2=G_{Rm}^2 \tag{4}$$

where: Y_m is the center of the circle in the Y_s plane, and G_{Rm} is the radius of the circle. Fukui used (4) to derive the minimum noise measure $M_{\min}$ for a given active device and the optimum source admittance Y_{om}.

Tucker [6] has corrected an error in Fukui's expression for G_{Rm} and has extended the work to cover circles of constant overall noise figure for the case of a preamplifier cascaded with a noisy main amplifier.

At microwave frequencies, it is preferable to carry out design work in the reflection coefficient plane because of the problems associated with admittance parameter measurements. The source admittance for the minimum noise measure (Y_{om}) obtained by the preceeding analysis can be readily converted to a reflection coefficient [5], but circles of constant noise measure in the reflection coefficient plane cannot be obtained from the results of

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admittance plane calculations. An alternative analysis is therefore needed.

An approach to the theory of noise in microwave devices based solely on reflection coefficients was first proposed by Penfield [7], who introduced the concept of noise power wave variables analogous to the power waves of scattering parameter theory [10]. Meys [8] later applied this concept to the noise characterization of active devices. The reflection coefficient approach involves the use of the following expression for the noise figure of an active device [8]:

$$F = F_{\min} + 4r_n \frac{|\Gamma_s - \Gamma_{on}|}{\left(1-|\Gamma_s|^2\right)|1+\Gamma_{on}|^2} \tag{5}$$

where

$$r_n = \frac{R_n}{Z_0}.$$

The work of Penfield [7], Meys [8], and others does not deal with the question of noise measure and its variation with the source termination of the active device.

Eisenberg [9] has derived circles of constant noise measure in the source reflection coefficient plane. Although based primarily on reflection coefficients, Eisenberg's approach still requires the use of some immittance parameters, notably the parameter R_{eg}. Since such parameters are not provided by the manufacturers of present day transistors, the user must undertake further measurements or calculations before using Eisenberg's results.

The difficulty of relying on immittance parameters for noise measure calculations can be overcome by using an expression for the available gain of a two-port active device which has been given by Bodway [10]

$$G_a = \frac{|S_{21}|^2\left(1-|\Gamma_s|^2\right)}{\left(1-S_{22}|^2\right)+|\Gamma_s|^2\left(|S_{11}|^2-|\Delta|^2\right)-2\,\mathrm{Re}(\Gamma_s C_1)} \tag{6}$$

where

$$C_1 = S_{11} - S_{22}^*\Delta$$
$$\Delta = S_{11}S_{22} - S_{12}S_{21}$$

Using (1), (5), and (6), the noise measure in terms of the source reflection coefficient can be expressed as

$$M = \frac{(F_{\min}-1)\left(1-|\Gamma_s|^2\right)|S_{21}|^2|1+\Gamma_{on}|^2 + 4r_n|S_{21}|^2|\Gamma_s-\Gamma_{on}|^2}{|1+\Gamma_{on}|^2\left(|S_{21}|^2\left(1-|\Gamma_s|^2\right)-\left(1-|S_{22}|^2\right)-|\Gamma_s|^2\left(|S_{11}|^2-|\Delta|^2\right)+2\,\mathrm{Re}(\Gamma_s C_1)\right)}. \tag{7}$$

Equation (7) can be shown to represent circles in the source reflection coefficient plane described by the following:

$$|\Gamma_s|^2 + |\Gamma_m|^2 - \Gamma_s\Gamma_m^* - \Gamma_s^*\Gamma_m = \gamma_m^2. \tag{8}$$

The centers and radii of the constant noise measure circles are given by

$$\Gamma_m = \frac{M|1+\Gamma_{on}|^2C_1^* + 4r_n|S_{21}|^2\Gamma_{on}}{M|1+\Gamma_{on}|^2P + |S_{21}|^2(4r_n - W)} \tag{9}$$

$$\gamma_m = \frac{\sqrt{M^2M_a + MM_b + M_c}}{M|1+\Gamma_{on}|^2P + |S_{21}|^2(4r_n - W)} \tag{10}$$

where

$$P = |S_{21}|^2 + |S_{11}|^2 - |\Delta|^2$$
$$Q = |S_{21}|^2 + |S_{21}|^2 - 1$$
$$W = |1+\Gamma_{on}|^2(F_{\min}-1)$$
$$M_a = |1+\Gamma_{on}|^4\left(PQ + |C_1|^2\right)$$
$$M_b = |1+\Gamma_{on}|^2|S_{21}|^2\big(8r_n\,\mathrm{Re}(\Gamma_{on}C_1) - \left(4r_n|\Gamma_{on}|^2 + W\right)P - (W-4r_n)Q\big)$$
$$M_c = |S_{21}|^4\left(W - 4r_n\left(1-|\Gamma_{on}|^2\right)\right).$$

The value of the minimum noise measure can be found by considering the noise measure circle of zero radius, i.e., set γ_m equal to zero in (10). This results in

$$M_{\min} = \frac{-M_b \pm \sqrt{M_b^2 - 4M_aM_c}}{2M_a}. \tag{11}$$

Equation (11) yields the same value of $M_{\min}$ as would have been obtained by using the immittance parameter equation given by Fukui [5] and Tucker [6], and can, therefore, be considered as the reflection coefficient plane analog of Fukuis expression. The elimination of the need for the parameter R_{eg}, however, results in a considerable simplification when compared with the earlier approach [5], [6].

The value of the minimum noise measure is taken as the smallest nonnegative value of $M_{\min}$ given by (11). The source reflection coefficient which results in the minimum noise measure can now be obtained by employing (9)

$$\Gamma_{om} = \frac{M_{\min}|1+\Gamma_{on}|^2C_1^* + 4r_n|S_{21}|^2\Gamma_{on}}{M_{\min}|1+\Gamma_{on}|^2P + |S_{21}|^2(4r_n - W)}. \tag{12}$$

The output reflection coefficient of the device, when Γ_{om} is presented to the input port, is given by [10]–[12]

$$\Gamma_{out} = \frac{S_{22} - \Delta\Gamma_{om}}{1 - S_{11}\Gamma_{om}}. \tag{13}$$

III. Experimental Amplifier

A single-stage GaAs MESFET microwave amplifier was designed for the minimum noise measure operation using the equations described in this paper. The circuit used a packaged type NE 71083 GaAs FET and was designed for a center frequency of 10 GHz. The s-parameters of the transistor in the common source configuration were measured, with a 50-Ω reference impedance, to be

$$S_{11} = 0.724\,\angle 46 \qquad S_{12} = 0.716\,\angle -47$$
$$S_{21} = 1.303\,\angle -106 \quad S_{22} = 0.616\,\angle 64$$
$$\times(V_{ds} = 3.0\mathrm{V},\quad I_d = 8\ \mathrm{mA}).$$

The stability of this device was evaluated using Woods stability criteria [12] which can be stated as follows: For unconditional stability

$$K > 1$$
$$|\Delta| < 1 \tag{14}$$

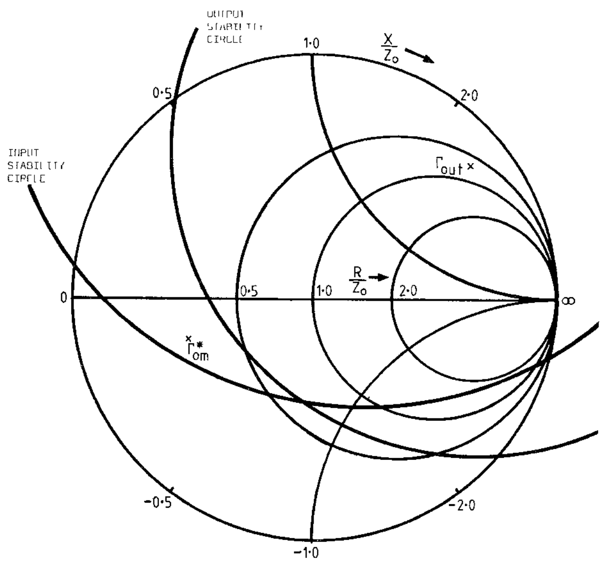

Fig. 2. Stability circles and matching terminations for the experimental amplifier plotted on the input and output reflection coefficient planes.

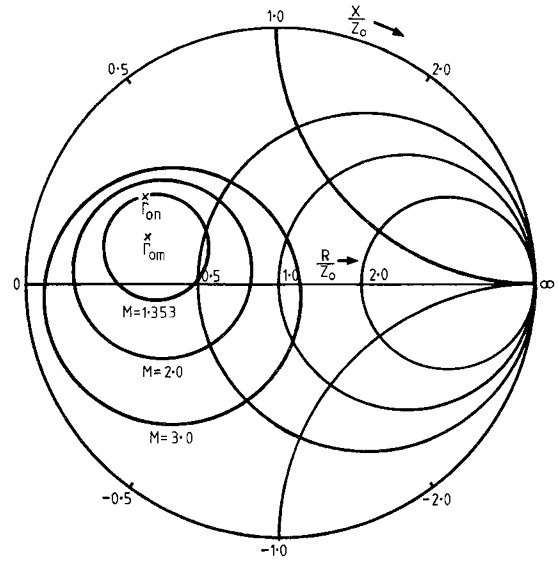

Fig. 3. Circles of constant noise measure and source matching terminations for the NE 70083 GaAs MESFET plotted on the source reflection coefficient plane.

where

$$K = \frac{1-|S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|}.$$

Using the measured s-parameters, it can be shown that the device is potentially unstable since $K = 0.681$ and $|\Delta| = 1.080$. For potentially unstable devices, regions of stable terminations can be mapped out on the input and output reflection coefficient planes by the use of stability circles [10].

The input and output stability circles for this transistor, under the stated conditions, are shown in Fig. 2.

The following noise parameters were supplied by the manufacturer of the FET:

$$F_{min} = 1.7 \text{ dB}$$
$$\Gamma_{on} = 0.620 \angle 148°$$
$$R_n = 12\ \Omega.$$

Using (11), the minimum noise measure obtainable with this device was calculated to be 1.303. Equation (12) yielded the value of the associated source reflection coefficient Γ_{om} to be $0.554\angle 161°$, which agrees very closely with the value obtained using Fukui's admittance plane equations [5].

Fig. 3 shows circles of constant noise measure on the source reflection coefficient plane, plotted using (9) and (10). It can be seen that if the device were matched for minimum noise figure, then the resulting noise measure would be 1.353.

The output reflection coefficient Γ_{out} of the transistor with Γ_{om} presented to its input port was then calculated from (13) to be $\Gamma_{out} = 0.849\angle 40°$.

Fig. 2 shows Γ_{om}^* and Γ_{out} plotted on the input and output reflection coefficient planes, respectively. Both values lie inside their respective stable regions indicating that a stable amplifier circuit is realizable. Quarter-wave transformers were used as input and output matching networks to transform the terminating impedances to 50 Ω. The amplifier circuit that resulted from this design procedure is shown schematically in Fig. 4.

The circuit was implemented using microstrip techniques on a 0.635-mm-thick "alumina" substrate and was evaluated using the HP 8410C network analyzer and the HP8970A automatic noise figure meter. The gain, noise figure, and noise measure response of the designed amplifier without any additional adjustment are shown in Fig. 5. It can be seen that the gain reached a maximum of 8.7 dB at 9.7 GHz, and a minimum noise figure of 4.0 dB was obtained at 9.2 GHz. The minimum noise measure obtained was 2.3 at a frequency of 9.7 GHz. The effective center frequency of the unadjusted amplifier was, therefore, within 3 percent of its design value, a discrepancy which was probably due to errors in the measurement of the transistors s-parameters. The minimum experimental noise measure was obtained at this effective center frequency.

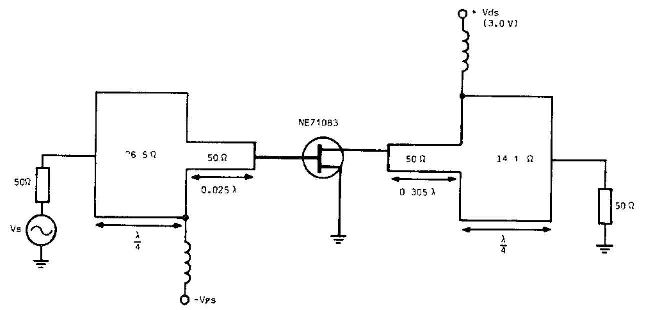

Fig. 4. Schematic representation of the completed experimental amplifier

The measured value of M_{min} was about twice that predicted by (11). It was suspected that this was due to discrepancies between the actual noise parameters of the device and those claimed for it by the manufacturer. Error analysis was therefore carried out on the value of noise measure obtained from (7), with respect to the device noise parameters, assuming only one parameter at a time was subject to errors. This revealed that the observed discrepancy required a 50 percent error in F_{min}, a 53 percent error in the modulus of Γ_{on}, or a 21-percent error in the phase angle of Γ_{on}. The value of noise measure obtained from (7) was found to be largely insensitive to errors in R_n, since a 50-percent error in this parameter resulted in only a 1-percent change in the value of M.

In the event of all the noise parameters being subject to errors simultaneously, the observed discrepancy in the value of M was obtained when all the parameters were in error by 16 percent.

It is therefore concluded that the noise measure equations derived here exhibit most sensitivity to errors in the phase of Γ_{on}.

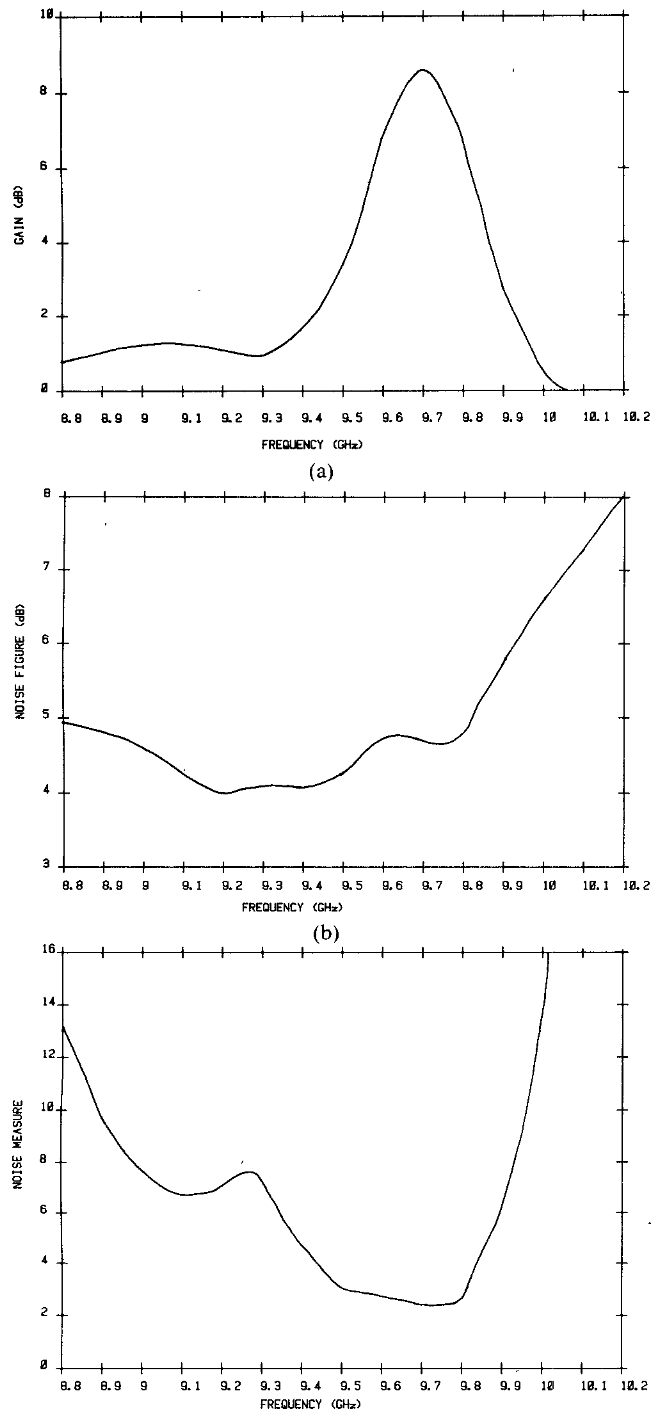

Fig. 5. Measured performance of the experimental amplifier. (a) Gain versus frequency. (b) Noise figure versus frequency. (c) Noise measure versus frequency.

This fact, combined with the difficulty of accurately measuring the phase of reflection coefficients, suggests that the discrepancy between predicted and measured noise measure is reasonable.

IV. Conclusion

The loci of constant noise measure presented here provide a useful graphical representation of how this quantity varies with source reflection coefficient (e.g., Fig. 3). The equations describing these circles have also been used to determine the minimum noise measure and associated optimum source termination for a given active device. The advantages of the new equations, when compared with existing methods of minimizing the noise measure, lie in the fact that they utilize reflection coefficient parameters which are readily available for most modern transistors. Furthermore, they yield results which are consistent with Smith Chart design techniques, thereby allowing graphical comparisons to be made with other criteria such as gain and noise figure.

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Graph Transformations of Nonuniform Coupled Transmission Line Networks and Their Application

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***Abstract* —The graph transformation method of [5] has been extended to apply for a class of coupled nonuniform transmission lines whose self and mutual line constants have the same functional dependence along the**

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